

# How does the Hubble Sphere limit our view of the Universe?

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## ABSTRACT

It has recently been claimed that the Hubble Sphere represents a previously unknown limit to our view of the universe, with light we detect today coming from a proper distance less than this “Cosmic Horizon” at the present time. By considering the paths of light rays in several cosmologies, we show that this claim is not generally true. In particular, in cosmologies dominated by phantom energy (with an equation of state of  $\omega < -1$ ) the proper distance to the Hubble Sphere decreases, and light rays can cross it more than once in both directions; such behaviour further diminishes the claim that the Hubble Sphere is a fundamental, but unrecognised, horizon in the universe.

**Key words:** cosmology: theory

## 1 INTRODUCTION

The existence of several cosmological horizons neatly carves up the space-time of a universe, with the *particle horizon* containing all of the events that a particular observer could ever have causal influence, and the *event horizon* containing all events that could ever causally influence that observer (Rindler 1956). The presence and extent of these cosmological horizons depends upon the evolution of the universal expansion, and hence ultimately on the mass-energy content of the universe (e.g. see Harrison 1993).

Recently, there have been claims of the existence of another, previously unrecognised horizon, dubbed the “Cosmic Horizon” and that this fundamentally limits our view of the Universe (Melia 2007; Melia & Abdelqader 2009; Melia 2009; Melia & Shevchuk 2012). In a spatially flat universe, this “Cosmic Horizon” is exactly the same as the well understood Hubble Sphere, the distance at which the universal expansion results in objects moving at the speed of light relative to us (Harrison 1991). For the sake of clarity, we will henceforth assume that the universe is spatially flat and refer to the “Cosmic Horizon” as the Hubble Sphere throughout this contribution.

In a previous paper, we demonstrated as being incorrect the claims that the Hubble Sphere sets a limit on what we can observe in the universe<sup>1</sup> (van Oirschot, Kwan, & Lewis

2010). However, Bikwa, Melia, & Shevchuk (2011) have reiterated these previous claims, considering photon paths in an expanding universe and stating that photons that we receive now are always from a *proper distance* which is less than the present size of the Hubble Sphere. In this paper, we consider this claim and show that that is not generically true. In fact, it is possible to show that a photon may cross the Hubble Sphere more than once in both directions, further revealing that its exalted status as a previously unrecognised “Cosmic Horizon” is still incorrect.

In Section 2 we discuss the key aspects of the evolution of the Hubble Sphere, and demonstrate how its size at the present time is not necessarily the limit of what we can see. We present the conclusions in Section 3. Throughout this paper, we will consider universes described by the Friedmann-Robertson-Walker metric.

## 2 THE EVOLUTION OF THE HUBBLE SPHERE

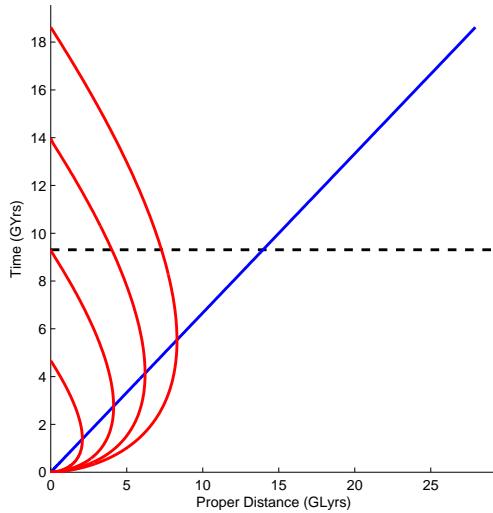
### 2.1 The Hubble Sphere

A consequence of the Hubble law is that objects at sufficiently large proper distance must be receding from us at velocities greater than the speed of light. The boundary between sub- and super-luminal recession velocities is a spherical surface around us, known as the Hubble Sphere, and,

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<sup>1</sup> Just as Lemaître (1925) realized that the apparent horizon in the de Sitter metric in static form arises from a bad choice of co-

ordinates, we showed that the apparent “Cosmic Horizon” originates from the reintroduction of these static coordinates.



**Figure 1.** The evolution of the Hubble Sphere (blue line) in an Einstein-de-Sitter universe. The red lines correspond to photon paths (null geodesics), while the black dashed line is the present age of the universe (assuming  $H_o = 70 \text{ km/s/Mpc}$ ).

by setting the recession velocity to the speed of light,  $c$ , into the Hubble law, is today at a proper distance of

$$R_h = \frac{c}{H_o} \quad (1)$$

where  $H_o$  is the present value of the Hubble constant.

In a generally evolving universe, the Hubble constant will be a function of time, and hence  $R_h$  is also a function of time. In a spatially flat universe with a single component of cosmic fluid with equation of state,  $\omega$ , it is straight-forward to show that the Hubble Sphere evolves as

$$\dot{R}_h = \frac{3}{2} (1 + \omega) c \quad (2)$$

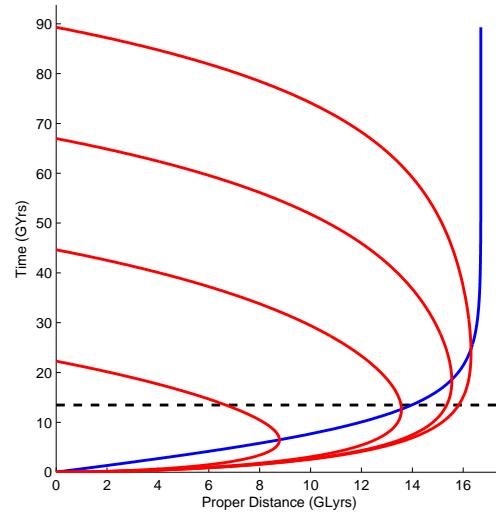
where the derivative is with respect to cosmic time (Melia 2009). In such a universe,  $R_h$  clearly evolves at a constant rate.

## 2.2 Expanding Hubble Spheres

An example of a universe governed by Equation 2 is presented in Figure 1, showing the evolution of  $R_h$  (blue line) over cosmic time for an Einstein-de-Sitter universe (spatially flat, containing only matter, so  $\omega = 0$ ). The horizontal dashed line corresponds to the present age of the universe (assuming  $H_o = 70 \text{ km/s/Mpc}$ ).

The red lines in Figure 1 correspond to photon paths from the Big Bang (at the origin) to us at various epochs of cosmic time; note, therefore, that these figures are essentially the same as those in Bikwa, Melia, & Shevchuk (2011), but reoriented, and showing both multiple light paths and the evolution of  $R_h$ . The claim by these authors is that  $R_h$  today (where the blue and black dashed lines intersect,  $\sim 14 \text{ GLyrs}$ ) is larger than the maximum proper distance achieved by a photon arriving at us today (roughly half that distance). Looking at the continually increasing value of  $R_h$  into the future, and the paths of photons received into the future, this appears to be true.

It is important to understand, however, what Figure 1 is



**Figure 2.** As Figure 1, but for a concordance cosmological model, with  $\Omega_m = 0.3$ ,  $\Omega_\omega = 0.7$  and an equation of state of dark energy of  $\omega = -1$  (i.e. Einstein's cosmological constant).

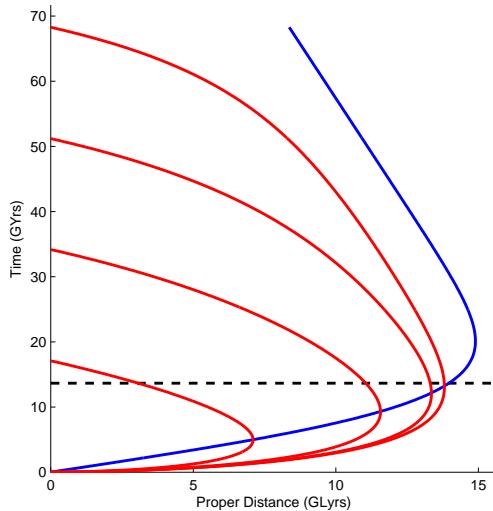
really telling us. In terms of proper distance, photons travel away from us at the Big Bang out to a maximum distance, before turning around and travelling back to the origin. The point at where the photon turns around in its journey is precisely where it crosses  $R_h$ ; this makes intuitive sense as it can be envisaged that, due to universal expansion, this is the point where the photon is effectively at rest with respect to us (this is shown rigorously in Section VIII of Ellis & Rothman (1993)).

The currently favoured cosmological model is constrained by multiple observations and contains a mix of cosmic fluids, being comprised of  $\sim 30\%$  matter and  $\sim 70\%$  dark energy with an equation of state,  $\omega \sim -1$  (e.g. Spergel et al. 2003). The evolution of  $R_h$  in such a universe is not simply described by Equation 2, but at early epochs, when the universe was matter-dominated (with  $\omega = 0$ ) we expect an evolution similar to an Einstein-de-Sitter universe, whereas at later times, the universe becomes dark energy dominated. If the equation of state of dark energy is  $\omega = -1$  (a cosmological constant), Equation 2 reveals that  $R_h$  is at a fixed proper distance from us.

Figure 2 presents the evolution of  $R_h$  in this universe, possessing the expected forms at early and late times, with a transition period (which we are now in)<sup>2</sup>. The behaviour of light rays in this cosmology is not too dissimilar to that presented in Figure 1, with light rays traveling outwards from the Big Bang, before turning back as they cross  $R_h$ . A key difference, however, is at late times where  $R_h$  asymptotes to a fixed distance from us, so that light rays will spend more and more time changing direction and moving back to the observer; it is at these later times that the Hubble Sphere coincides with the event horizon, truly limiting what we can see.

Again, the argument made by

<sup>2</sup> We note that such a figure is not new, and an interested reader is invited to examine the excellent depiction of this cosmology in Figure 1 of Davis & Lineweaver (2004), presenting key features of the universe in several coordinate systems.



**Figure 3.** As Figure 2, but adopting an equation of state of dark energy to be  $\omega = -1.1$  (i.e. phantom energy).

Bikwa, Melia, & Shevchuk (2011) appears to hold, with photons arriving today not having travelled outside the present day Hubble Sphere. For future observers, this remains true with all photons travelling out from the Big Bang to  $R_h$  before heading back to the origin.

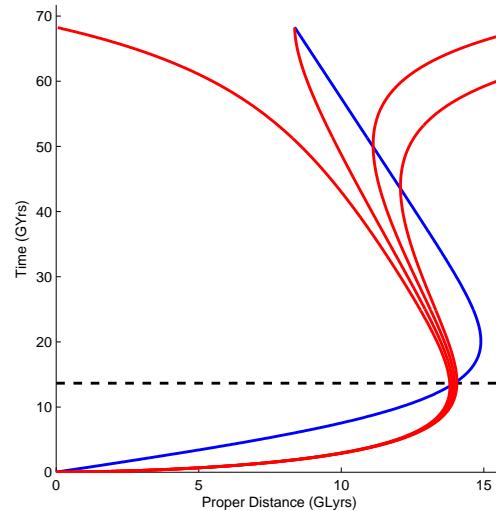
### 2.3 Collapsing Hubble Spheres

In the examples presented in Section 2.2,  $R_h$  continually expands, to infinity in Figure 1 and asymptotizing to a finite value in Figure 2. But does  $R_h$  have to expand? An examination of Equation 2 reveals that if the equation of state,  $\omega < -1$ , then  $\dot{R}_h$  can be negative; with such an equation of state, such a cosmic fluid is known as *phantom energy*. The presence of phantom energy has a dramatic effect on the expansion of the universe, potentially resulting in a cosmic doomsday where galaxies, planets and eventually atoms are ripped apart by the accelerating expansion (e.g. Caldwell, Kamionkowski, & Weinberg 2003).

Figure 3 presents the evolution of  $R_h$  in a universe with a present day matter density of  $\Omega_m = 0.3$  and dark energy density of  $\Omega_\omega = 0.7$ , and equation of state of dark energy of  $\omega = -1.1$ . During the earlier matter-dominated stage of the universe, the behaviour is similar to that seen in Figure 2, but as the universe becomes dark energy dominated,  $R_h$  reaches a maximum extent and then begins to decrease.

Examining the photon paths in Figure 3 reveals a similar behaviour to the previous figures, with photons heading out from the Big Bang before turning back towards the origin, with the turning point being when the photons cross  $R_h$ . Again, photons we receive today turn around at a distance less than  $R_h$  today, as proposed by Bikwa, Melia, & Shevchuk (2011). However, it is clear that observers in the distant future receive photons that turned around in their journey at a proper distance substantially larger than  $R_h$  at the time the photon is received; this directly contradicts the ideas proposed by Bikwa, Melia, & Shevchuk (2011).

Finally, in Figure 4 we further examine this phantom energy cosmology by presenting photon paths that do not



**Figure 4.** As Figure 3, but considering photon paths that do not necessarily reach the origin.

necessarily arrive back at the observer at the spatial origin. As in the previous figures, light paths move out from the Big Bang and turn back towards the observer by passing through  $R_h$ . While one of the photon paths arrives at the observer, the collapsing Hubble Sphere influences the remaining photon paths, with each of them encountering  $R_h$  for a second time (and again the photon can be thought of as at rest with respect to us), before heading to larger proper distance. The fact that such a photon path can pass through the Hubble Sphere multiple times in differing directions is another nail in the concept that the Hubble Sphere is a “Cosmic Horizon”.

## 3 CONCLUSIONS

In this letter, we have examined the evolution of the Hubble Sphere,  $R_h$ , over cosmic time, showing that its present size is not necessarily a limit on the maximum proper distance from which we are receiving photons at the present time, contrary to the claims recently made in the literature (Bikwa, Melia, & Shevchuk 2011).

It should be remembered that the Hubble Sphere is not a complex concept, and as demonstrated here (and in Ellis & Rothman (1993)), as well as being the boundary between sub- and super-luminal expansion in the universe, it represents the inflection points on a photons path between the Big Bang and an observer (when viewed in terms of proper distance and cosmic time).

The evolution of  $R_h$  depends ultimately on the mass-energy content of the universe. In universes like our own, which have so far been matter dominated for much of their history,  $R_h$  initially evolves like an Einstein-de-Sitter universe, and as  $R_h$  keeps growing, it is trivial to say that the proper distance to the turning point of a photon, which is equal to  $R_h$  at the time of turning, is smaller than the Hubble Sphere now. For universes with a different mix of cosmic fluids, or those which components that evolve, such a statement cannot be necessarily made.

We finally reiterate that photons can cross the Hub-

ble Sphere multiple times, and one could imagine a universe with an evolving dark energy component that oscillates between matter and phantom energy. With such a universe, the Hubble Sphere could also oscillate in and out, with a photon path from the Big Bang traversing Hubble Sphere multiple times before reaching an observer. If our inflationary epoch was driven by phantom energy (e.g. Capozziello, Nojiri, & Odintsov 2006), this may have already happened. Hence the Hubble Sphere is not a “Cosmic Horizon”.

None of this should really come as a surprise, as the evolution of the particle and event horizons, and the Hubble Sphere have been the focus of several classic papers (e.g. Rindler 1956; Harrison 1991; Ellis & Rothman 1993). Recent contributions have added little to our understanding.

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